

511 keV Line Emission from Nearby Spherical Dwarf Galaxies

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Abstract. The observed galactic 511 keV line has been interpreted in a number of papers as a possible signal of dark matter annihilation within the galactic bulge. If this is the case then we should expect a similar spectral feature associated with nearby dwarf galaxies which are dark matter dominated. It has recently been argued [1] that the absence of such a signal excludes a dark matter explanation as the major source for the galactic 511 keV line. In the model presented here dark matter in the form of heavy quark nuggets produces the galactic 511 keV emission line through interactions with the visible matter. It is argued, however, that this type of interaction is not subject to the strong dark matter annihilation constraints presented in [1].

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Contents

1	Introduction	1
2	Quark nugget dark matter	2
3	The galactic 511 line	3
4	Dwarf Galaxies	5
5	Conclusion	8

1 Introduction

The origin of the galactic 511 keV positron annihilation line remains an outstanding problem in galactic astrophysics in terms of both morphology and intensity. Some details of this feature of the galactic spectrum and of its history can be found in the review [2]. Of primary interest is the low disk-to-bulge intensity ratio of the 511 keV emission and the apparently low injection energy of the positrons responsible. The absence of a clear astrophysical source combined with the largely spherical morphology lead to the suggestion that the primary source of the required population of low energy positrons could be the annihilation of light dark matter. For the details of this argument see the original paper [3] and the many related references for followup papers found in the review article [2]. The models reviewed in [2] include a number of models in which the low energy positrons responsible for the 511 keV line are produced in the annihilation of light weakly interacting massive particles (WIMPs) and it is this class of models to which the analysis of [1] is most readily applicable.

Following the detection of a stronger than expected galactic 511 keV line it was noted [4] that the dwarf satellite galaxies of the Milky Way offer a test of its potential dark matter origin. This suggestion motivated the analysis of [1] in which it was shown that observations from the SPI spectrometer on INTEGRAL of nearby dwarf galaxies contradict the basic idea that the the galactic 511 keV line could be related to dark matter annihilation. In a purely J-factor based analysis a dark matter generated galactic 511 keV line would require similar emission from the dwarf galaxies at levels significantly above those observed.

We consider an alternative dark matter based interpretation of the galactic 511 keV line consistent with both the presence of a disk component in the galactic line's morphology and with the non-detection of 511 keV emission from the dwarf satellite galaxies. This model [5, 6] was originally proposed to offer a natural explanation of the observed relation between the dark and visible components of the cosmic energy density $\Omega_{\text{dark}} \sim \Omega_{\text{visible}}$, in contrast most other proposals considered in this context have been specifically designed to produce the observed galactic 511 keV line. We will review the basic properties of this model in Section 2. For now it is sufficient to mention that in this model the dark matter consists of macroscopically large composite objects composed of light standard model quarks or antiquarks. These "nuggets" of antiquarks are surrounded by a layer of low energy positrons which, if these objects serve as the dark matter, will occasionally annihilate with the electrons of the interstellar medium and contribute to the observed emission [7, 8]. In this case the strength of the 511 keV emission is dependent on both the visible n_{visible} and dark matter n_{DM} densities,

which is in huge contrast with conventional dark matter models in which the annihilation rate is determined by the dark matter density n_{DM} only. From the reliance of emission on the local visible matter density in this model it is obvious that the dwarf satellite galaxies should be weak 511 keV sources as they are relatively poor in interstellar matter.

Following a brief review of the quark nugget dark matter model in section 2 we will provide an analysis of galactic 511 keV line emission within the context of this model in section 3. Finally, in section 4 we will use the 511 keV line strength from the galactic centre to extrapolate to emission from nearby dwarf spherical galaxies and demonstrate that the expected emission is fully consistent with present constraints. Section 5 is our conclusion.

2 Quark nugget dark matter

In this section we provide a brief overview of the quark nugget dark matter model. Some further details on observational constraints are provided in the short review paper [9] and the references therein. Details of a possible formation mechanism for the quark and antiquark nuggets is provided in the recent paper [10], see also [11] with a short overview on the formation stage of the nugget’s evolution during the QCD transition in early Universe.

The idea that the dark matter may take the form of massive composite objects composed of standard model quarks in a novel phase goes back to stranglet models [12]. However the model we consider here (originally formulated in [5, 6]) has some important distinctions from strangelet dark matter. First of all, the nuggets are formed of matter as well as antimatter as a result of separation of charges. Secondly, the stability of the DM nuggets is provided by extra pressure generated by axion domain walls, in contrast with strangelets for which stability is assumed to be achieved even in vacuum, at zero external pressure. Finally, an overall coherent baryon asymmetry in the entire Universe results from the strong CP violation due to the fundamental θ parameter in QCD which is assumed to be nonzero at the beginning of the QCD phase transition.

Unlike conventional dark matter candidates such as WIMPs the dark matter/antimatter nuggets are strongly interacting but macroscopically large. They do not contradict known observational constraints on dark matter or antimatter for three main reasons [13]:

- They carry a huge (anti)baryon charge $|B| \gtrsim 10^{25}$, and so have an extremely tiny number density;
- The nuggets have nuclear densities, so their effective interaction is small $\sigma/M \sim 10^{-10} \text{ cm}^2/\text{g}$, well below the typical astrophysical and cosmological limits which are on the order of $\sigma/M < 1 \text{ cm}^2/\text{g}$;
- They have a large binding energy such that baryon charge in the nuggets is not available to participate in big bang nucleosynthesis (BBN) at $T \approx 1 \text{ MeV}$.

To reiterate: the weakness of the visible-dark matter interaction is achieved in this model due to the small geometrical parameter $\sigma/M \sim B^{-1/3}$ rather than due to the weak coupling of a new fundamental field with standard model particles. In other words, this small effective interaction $\sim \sigma/M \sim B^{-1/3}$ replaces a conventional requirement of sufficiently weak interactions of the visible matter with WIMPs.

While the observable consequences of this model are on average strongly suppressed by the low number density of the quark nuggets the interaction of these objects with the visible matter of the galaxy will necessarily produce observable effects. Any such consequences will

be largest where the densities of both visible n_{visible} and dark matter n_{DM} are largest such as the core of the galaxy. In other words, the nuggets behave as conventional cold dark matter in the environment where the visible matter density is small, while they become interacting and radiation emitting objects (i.e. effectively become visible matter) when in an environment of sufficiently large density.

The rate of annihilations between visible matter and antiquark nuggets is proportional to the product of the local visible and DM distributions at the annihilation site. The observed flux for any form of annihilation driven emission thus depends on the same line-of-sight integral

$$\Phi \sim R^2 \int d\Omega dl [n_{\text{visible}}(l) \cdot n_{\text{DM}}(l)], \quad (2.1)$$

where $R \sim B^{1/3}$ is a typical size of the nuggets which determines the effective interaction cross section between the dark and visible matter. As $n_{\text{DM}} \sim B^{-1}$ the effective interaction is suppressed by a factor of $\sim B^{-1/3}$ as mentioned above. The form of expression 2.1 should be contrasted with the J-factor line-of-sight integral used in [1] and many other analysis of dark matter related emission,

$$J \equiv \int_{\Delta\Omega} d\Omega \int \rho_{\text{DM}}^2 dl \quad (2.2)$$

which depends strictly on the dark matter density and thus characterizes dark matter-dark matter interactions along a given line of sight. The fact that expression (2.1) mixes the dark and visible matter distributions is important to the arguments which follow. The dependence on the more strongly clumped visible matter density make the flux predictions of this model more difficult despite the seemingly simple dependance on a single parameter $\langle B \rangle$. The estimation $\langle B \rangle \sim 10^{25}$ can be fixed by assuming that the annihilation of electrons from the visible matter with the positrons from the antiquark nuggets saturates the observed galactic 511 keV line [7, 8]. It has also been assumed here that the observed dark matter density is saturated by the combination of quark and antiquark nuggets.

The annihilation of galactic matter within an antiquark nugget is a complex many body problem which involves a range of annihilation channel each giving rise to emission in a different frequency band. Emission strengths in different bands are expressed in terms of the same integral (2.1), and therefore, the relative intensities are completely determined by the internal structure of the nuggets which is described by conventional nuclear physics and basic QED. Estimates of the nugget contribution to the diffuse galactic spectrum can be found in the original references [7, 8, 14–18], where predictions of the model have been confronted with observations in specific frequency bands covering more than eleven orders of magnitude, from radio frequency with $\omega \sim 10^{-4}$ eV to γ rays with $\omega \sim 10$ MeV. It has been shown that there are no contradictions with available data. Furthermore, there are a number of frequency bands where some excess emission is observed, but not explained by conventional astrophysical sources. It has been argued that the contribution of emission from the antiquark nuggets may explain, either wholly or in part these observed excesses in the galactic diffuse emission, see the original works [14–18] for details.

3 The galactic 511 line

Here we present a simplified analysis of the galactic 511 keV line strength in the context of the quark nugget dark matter model. The annihilation of visible matter with the nuggets is a well understood process entirely grounded in known QED and QCD physics, and these

processes will operate identically in a wide variety of astrophysical environments. In the outer regions of the electrosphere of an antiquark nugget the positrons carry velocities at the thermal (eV) scale and low energy galactic electrons entering the electrosphere will rapidly form positronium bound states [8]. One quarter of positronium decays result in the production of a narrow 511 keV line while the remaining fraction contribute to a three photon continuum also observed from the galactic centre.

The rate at which matter annihilates with the nuggets to produce a 511 keV photon pair in a given environment is approximately given by,

$$\frac{dN}{dt dV} = f_{511} \frac{\sigma}{M} \rho_{\text{DM}} \langle v \rangle n_e \quad (3.1)$$

where f_{511} is the probability that a collision results in the emission of a 511 keV photon, σ/M is the average cross-section to mass ratio of the nuggets, $\rho_{\text{DM}} \simeq n_{\text{DM}} M$ is the dark matter mass density, $\langle v \rangle$ is the averaged relative velocity between the nuggets and the visible matter and n_e is the number density of electrons (both free and bound) in the interstellar medium. As we are interested in the relative strength of 511 keV emission from the Milky Way and other nearby galaxies we will define the coefficient κ to stand in for the unknowns in the microscopic details of the nuggets.

$$\frac{dN}{dt dV} \equiv \kappa \rho_{\text{DM}} \langle v \rangle \rho_{\text{ISM}} \quad (3.2)$$

where ρ_{ISM} is the mass density of the interstellar medium¹. The total annihilation rate of positrons within the galactic centre is estimated to be $\Gamma_{e+} = 2 \times 10^{43} \text{ s}^{-1}$ which allows us to fix the value of the coefficient κ to be,

$$\kappa = \frac{\Gamma_{e+}}{\int dV \rho_{\text{DM}} \langle v \rangle \rho_{\text{ISM}}} \quad (3.3)$$

The integral appearing in equation (3.3) should run over the matter distribution of the galactic centre which is rather complicated. As a first simplified estimate consider the case where the dark matter has a roughly constant density core out to the kiloparsec scale and the collisional velocity is fixed around the typical galactic scale $v_g \approx 200 \text{ km/s}$. In this case the integral runs over only the visible matter distribution and we have,

$$\kappa = \frac{\Gamma_{e+}}{\rho_{\text{DM}} v_g M_{\text{ISM}}} \quad (3.4)$$

Where M_{ISM} is the total mass of interstellar gas within the galactic centre. Note that a more strongly cusped dark matter halo would increase the value of the integral in the denominator of expression (3.3) and result in a lower estimate of κ . Such an estimate would represent an increased suppression of 511 keV emission and sets an upper limit on emission per nugget and allows us to estimate the maximum 511 keV flux expected from nearby dwarf galaxies².

¹It should be noted that stellar matter does not contribute significantly to the production of 511 keV photons as photons produced within a star will be rapidly absorbed.

²A more careful treatment of the matter distribution of the galactic centre would allow us to better constrain the emission from nuggets within the Milky Way but is not necessary for present purposes as even this relatively high estimate of nugget emission will be shown to be consistent with constraints from the dwarf galaxies.

In keeping with our intention to make a conservative estimate of the emission from nearby spherical dwarfs we will adopt a relatively low dark matter density of $\rho_{\text{DM}} \approx 1 \text{ GeV}/\text{cm}^{-3}$ and a galactic velocity of $v_g \approx 200 \text{ km/s}$. The visible matter mass in the galactic bulge is estimated to be on the order of $10^{10} M_\odot$ [19]. Using these estimates we arrive at a value for the emission coefficient of,

$$\kappa \approx 10^{-31} \frac{\text{cm}^2}{\text{GeV}^2} \quad (3.5)$$

4 Dwarf Galaxies

We now want to determine the 511 keV line signal associated with a dwarf galaxy. These objects are below the resolution of the SPI and will consequently appear as point sources in the INTEGRAL data. In this case we can write the total flux received from a galaxy at distance d as,

$$\frac{dN_{511}}{dt dA} = \frac{\kappa}{4\pi d^2} \int dV \rho_{\text{ISM}} v \rho_{\text{DM}} \quad (4.1)$$

with the integration running over the volume containing the galaxy. In order to extract flux estimates we must consider possible matter profiles for the dwarf galaxies³. As above we will attempted to make assumptions which produce conservative constraints (that is models which produce the largest possible flux). To this end we will assume that the dark matter of the dwarf galaxies follows a conventional NFW profile [20] which will produce stronger central intensity than alternative cored distributions,

$$\rho_{\text{NFW}}(r) = \frac{\rho_s}{r/r_s(1 + r/r_s)^2} \quad (4.2)$$

for a scale radius r_s and density ρ_s .

The situation is more complicated for the visible matter. The dwarf galaxies are known to contain very little gas [21] which is the visible matter component of most relevance to diffuse 511 keV emission. The stellar distribution in dwarf spheroidal galaxies has been modelled using a Plummer profile,

$$\rho = \rho_0 \left[1 + \left(\frac{r}{r_{1/2}} \right)^2 \right]^{-5/2} \quad (4.3)$$

where ρ_0 is the central density and $r_{1/2}$ is the half-light radius of the stellar population (see for example [23] for a discussion of different stellar profiles.) We will assume that what ISM components are present follow a similar profile and mass scale though in reality the gas component could be much smaller. Alternative matter distributions have also been considered including exponential profiles and tidally stripped King profiles which have been shown to provide a good fit to star counts [23]. It has also been suggested that a better fit to the Draco dwarf spheroid is obtained from a modified Plummer profile which falls as $\sim r^{-7/2}$ at large radii [22]. However the Plummer profile is known to provide an adequate fit to a large number of dwarf spheroids and the relatively strong central peak of the expression (4.3) should produce a correspondingly strong 511 keV emission line when convolved with the NFW profile as will its large spacial extent relative to the truncated King profile.

³Unfortunately a model independent comparison with the results of [1] is not possible due to the fact that the matter integral appearing in expression (4.1) contains both dark and visible matter factors as opposed to simply the J-factor, $J \sim \rho_{\text{DM}}^2$.

The mean velocity of the gravitationally bound matter will be assumed to be the virial velocity of the system.

$$v_{vir} = \sqrt{\frac{GM_{Dyn}}{2r_{1/2}}}. \quad (4.4)$$

where M_{Dyn} is the dynamical mass within a half-light radius. Using the matter profiles (4.2) and (4.3) for the dark and visible components respectively and assuming a virial velocity given by (4.4) the predicted flux can be written as,

$$\frac{dN_{511}}{dt dA} = \frac{\kappa}{d^2} v_{vir} \int \frac{\rho_0 \rho_s r_s r dr}{(1 + (r/r_{1/2})^2)^{5/2} (1 + r/r_s)^2} \quad (4.5)$$

Unlike WIMP annihilation models the 511 keV flux cannot be formulated purely in terms of the J-factor as it is also dependent on the visible matter profile. It is thus necessary for us to reformulate expression (4.5) in terms of observable quantities.

As the dwarf spheroids are dark matter dominated [23–25] we will take the dynamical mass within a half-light radius to be primarily due to dark matter. We will also assume that the length scale associated with the dark matter halo is much greater than that associated with the visible matter (ie. $r_s \gg r_{1/2}$). In this case we can integrate the NFW profile and express the dark matter scale density may be written as,

$$\rho_s r_s \approx \frac{M_{Dyn}}{2\pi r_{1/2}^2}. \quad (4.6)$$

The central density of the visible matter may be estimated by integrating the density profile in expression (4.3) and formulating the result in terms of the stellar mass within a half-light radius. This gives a central density of,

$$\rho_0 \approx \frac{3M_\star}{8\sqrt{2}\pi r_{1/2}^3}. \quad (4.7)$$

We may then represent expression (4.5) as,

$$\begin{aligned} \frac{dN_{511}}{dt dA} &= \frac{3\kappa}{16\sqrt{2}\pi^2 d^2} \frac{v_{vir} M_{Dyn} M_\star}{r_{1/2}^5} \\ &\times \int \frac{r dr}{(1 + (r/r_{1/2})^2)^{5/2} (1 + r/r_s)^2}. \end{aligned} \quad (4.8)$$

If the dark matter distribution is sufficiently large relative to the visible matter that we may neglect the long range behaviour of the NFW profile then the integration of expression (4.8) is particularly simple, and it is given by

$$\frac{dN_{511}}{dt dA} \approx \frac{\kappa M_{Dyn} M_\star}{32\pi^2 d^2 r_{1/2}^3} \sqrt{\frac{GM_{Dyn}}{r_{1/2}}}. \quad (4.9)$$

So that we may estimate the total flux from a dwarf spheroid based on its observed dynamical mass, stellar mass and half light radius. The required parameters for the majority of galaxies

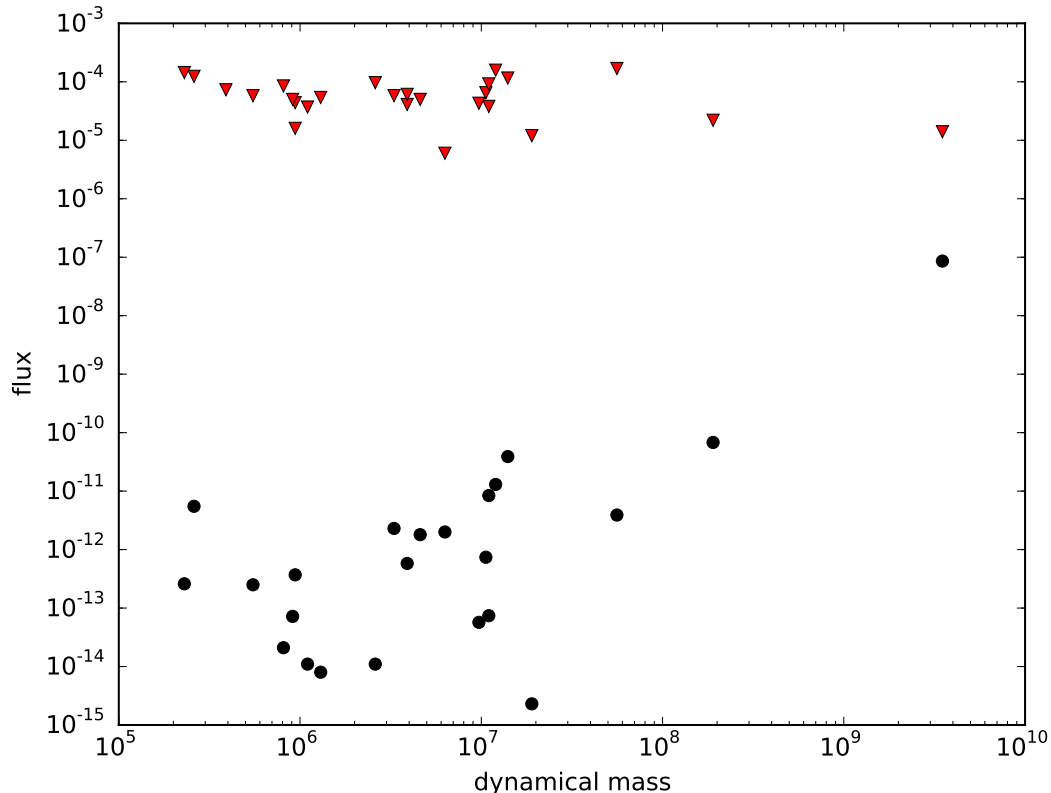


Figure 1. Total 511 keV photon flux (in $\text{cm}^{-2}\text{s}^{-1}$ predicted from the dwarf spheroid galaxies described in [26] as a function of dynamical mass. The red triangles represent the observed flux or its upper limit in cases where no significant 511 keV emission is detected. The black squares give the approximate 511 keV flux from a population of quark nuggets capable of providing the observed galactic 511 keV derived from expression (4.9).

considered in [1] may be found in [26]⁴. It is therefore possible to use expression (4.9) to predict a 511 keV line flux (normalized to that of the milky way) for each of the dwarf galaxies for which the relevant parameters are available.

Evaluating expression (4.9) for all the dwarf galaxies for which the relevant physical parameters are available in [26] results in the data shown in figure 1. As can easily be seen in the results presented in figure 1 the 511 keV flux generated by a population of quark nuggets capable of explaining the galactic 511 keV line will be well below the observational limits. This is due to the fact that the interaction rate of the nuggets is set by the product of their density with the local visible matter density. In this case the predicted emissivity does not scale simple with the J-factor and, despite being dark matter dominated, the dwarf spheroid galaxies do not present a particularly attractive target for this type of dark matter model.

Adopting the flux limits cited in [1] and the physical parameters of [26] allows us to translate the 511 keV flux limits into constraints on the parameter κ through equation (4.9).

⁴This sample does not include the Reticulum II dwarf which shows a relatively strong 511 keV line as well as several others included in the analysis of [1]. It does however cover a wide range of the known dwarfs and there is no reason to expect that it is not a sufficiently representative subgroup of the known dwarfs.

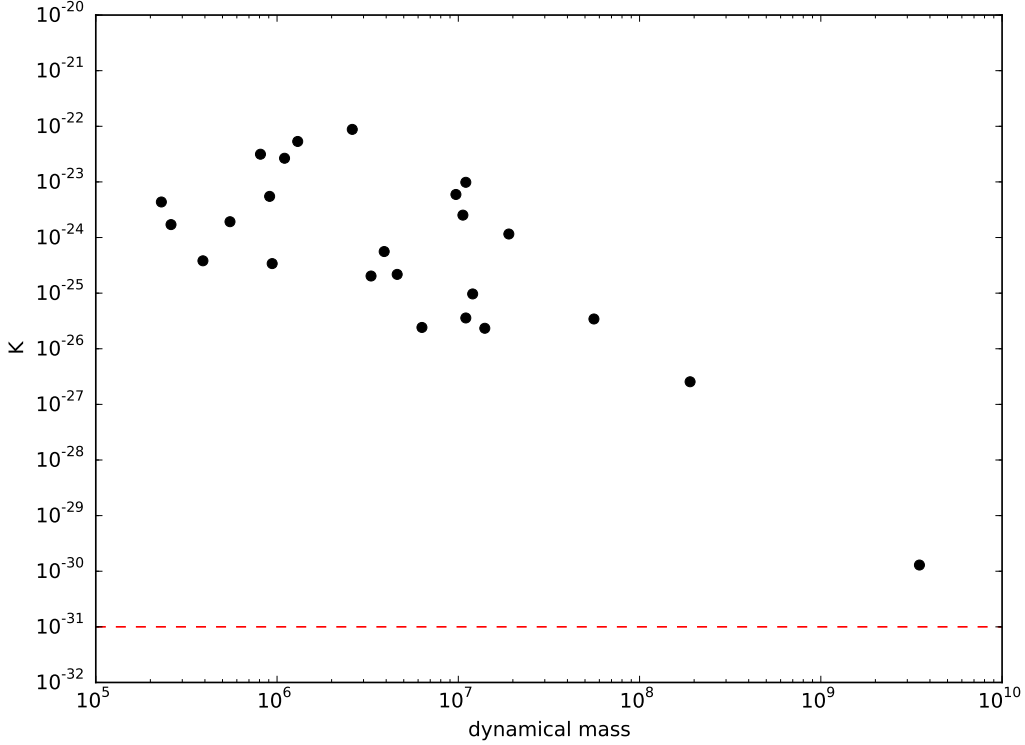


Figure 2. Black dots show the values of the 511 keV line emission coefficient (κ) extracted from each of the dwarf spheroidal galaxies considered in [1] and described in [26]. The value of κ predicted if emission from electron-positron annihilation within antiquark nuggets saturates the galactic 511 keV line is shown by a red dashed line.

Figure 2 shows the range of κ values extracted from the dwarf spheroidal galaxies as well as that obtained from the Milky Way. It is easily seen that the strength of the galactic 511 keV line produces a limit on the emission coefficient κ at least an order of magnitude at least an order of magnitude below the best possible constraints obtained from the dwarf spheroids.

Finally, it should be noted that, in this model, 511 keV emission from the dark matter dominated and gas poor dwarf galaxies is mainly limited by the lack of visible matter with which the nuggets can interact. As such, the correlation between visible brightness and 511 keV line strength inferred in [1] does not contradict a possible dark matter origin. This is in contrast to models in which 511 keV emission is driven purely by dark sector physics in which case the arguments of [1] can be used to argue against 511 keV emission from dark matter.

5 Conclusion

The model we have advocated here was originally invented as a simple and natural explanation of the observed relation: $\Omega_{\text{dark}} \simeq \Omega_{\text{visible}}$ by postulating that both elements originate from one and the same QCD epoch and proportional to one and the same Λ_{QCD} scale, see short review articles [9] regarding the observational constraints and [11] regarding the formation stage during the QCD transition. The immediate consequence of this proposal is the presence of

antimatter in the form of macroscopically large antiquark nuggets. An equal portion of matter and antimatter in our universe does not contradict the conventional and naive arguments on the nearly total observational absence of antimatter as has been explained in section 2.

This model has a single fundamental parameter, the mean baryon number of a nugget $\langle B \rangle$, which scales all observational consequences. If the value of $\langle B \rangle$ is sufficiently large the model is consistent with all known astrophysical, cosmological, satellite and ground based constraints as highlighted in section 2. Furthermore, in a number of cases the predictions of this model are very close to presently available limits, and very modest improvements on those constraints may lead to the discovery of the nuggets. Even more than that: there are a number of frequency bands where some excess of emission is observed, and this model may explain some portion, or even the entire excess of the observed radiation in these frequency bands.

In the present work we have explicitly demonstrated that this model is consistent with the non-detection of a 511 keV line from dwarf satellite galaxies. This should be contrasted with more conventional dark matter annihilation models for origin of the galactic 511 keV line (see the original paper [3] and review article [2]) which the analysis of [1] shows to contradict the observed levels of 511 keV emission from nearby dwarf spheroids. The crucial difference between our model and the large number of WIMP based models reviewed in [2] is that 511 keV emission strength is proportional to both the visible and dark matter densities as formula (2.1) states. In the case of 511 keV emission originating from the annihilation of light WIMP dark matter the positronium production rate is not sensitive to the visible matter density but depends exclusively on the dark matter J-factor according to eq. (2.2). Precisely this fundamental difference in morphological properties between the models determines the drastic variation in the e^+e^- annihilation rate.

One should also add that in our model the positrons fill the electrosphere of an antiquark nuggets with typical atomic velocities, such that the dominant mechanism of annihilation is through positronium formation in perfect agreement with observations. This should be contrasted with the other dark matter explanations of the 511 keV line reviewed in [2] when the typical positron energies are order of MeV rather than atomic (eV scale) energies. Furthermore, our model predicts that the morphology of the 511 keV line should not be perfectly spherical as the rate depends on the visible density distribution according to (2.1). These morphological features are also consistent with observations reviewed in [2].

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